MECHANICS.

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181. Proposed by F. ANDEREGG, Professor of Mathematics, Oberlin College, Oberlin, Ohio.

A triangle AOB, of which the sides, OA, AB, and the angle at O are a, b, and α, revolves uniformly about O, so that OA makes the angle nt with the axis of x, and carries a circle of which AB is the diameter. Prove that a point moving in the circumference of the carried circle with twice the angular velocity of the triangle will describe an ellipse whose axes are

$$\sqrt{(a^2+b^2+2ab\cos\alpha)}\pm\sqrt{(a^2+b^2-2ab\cos\alpha)}.$$

Solution by F. H. SAFFORD, Ph. D., The Universitv of Pennsylvania.

Let the third side of the the triangle be c and let the median from O be d, then

$$c=\tfrac{1}{2}\sqrt{(a^2+b^2-2ab\cos\alpha)},\quad d=\tfrac{1}{2}\sqrt{(a^2+b^2+2ab\cos\alpha)}.$$

Consider the triangle at the time when the moving point is on the produced median and take the x-axis as coincident with the position of the median at this time. By taking the rotation of the moving point in the direction *opposite* to that of the triangle and noticing that the circle is itself subject to the rotation of the triangle on which it is fixed, the double velocity of the point causes it to have with respect to the axis of x a rotation opposite in direction and equal in amount to that of the triangle. Hence the coördinates of the moving point are

$$x=d\cos\theta+c\cos\theta,\ y=d\sin\theta-c\sin\theta,\ \text{or}\ x=(d+c)\cos\theta,\ y=(d-c)\sin\theta,$$

from which follows the ellipse with axes as stated.

The problem may be executed mechanically as follows. At the point O fasten to the fixed plane a circle of twice the diameter of the carried circle. This circle is to remain fixed and to be connected with the carried circle by an *open* belt.

185. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

A perfectly flexible rope whose weight is w per linear unit, and length $2l$, rests in equilibrium on a smooth peg. If now one end be raised a distance a and then released, find the time in which this end will rise to the height x above its original position, and the tension at that instant of the rope at the point where it passes over the peg.

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Denote by z the height of one end above the initial position, and let m be the mass per unit of length. Then

$$2lm\frac{d^2z}{dt^2}=2mgz.\quad\therefore 2\frac{dz}{dt}\frac{d^2z}{dt^2}=\frac{2gz(dz/dt)}{l}.$$

$$\left(\frac{dz}{dt}\right)^2=\frac{g}{l}(z^2-a^2),\text{ since } z=a \text{ when } \frac{dz}{dt}=0.$$

$$\therefore t=\sqrt{\frac{l}{g}}\int\frac{dz}{\sqrt{(z^2-a^2)}}=\sqrt{\frac{l}{g}}\log\frac{z+\sqrt{(z^2-a^2)}}{a},$$

if $t=0$ when $z=a$. Putting $z=x$ we get the required time.

Also we have, [Tension at peg]δt=momentum generated by tension in time $\delta t=mv\delta t.v=mv^2\delta t$.

$\therefore$ Tension at peg$=m\left(\frac{dx}{dt}\right)^2=\frac{mg}{l}(x^2-a^2)$, when $z=x$.

Also solved by S. A. Corey, and G. B. M. Zerr.

MISCELLANEOUS.

152. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

A conductor, the equation of the surface of which is

$$\frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{9}=1,$$

is charged with 80 units of electricity, what is the density at a point for which $x=3$, $y=3$? If the density of this point be a, what is the whole charge on the ellipsoid? [From Peirce's *Potential Functions*, example 165, p. 388.]

Solution by M. E. GRABER, A. M., Tiffin, Ohio.

The mass of an ellipsoidal shell is $\frac{4}{3}\pi\rho d(abc)=4\pi Mabc$, and $Q=A4\pi\mu abc\rho$. $s=A\mu\theta$ and $\theta=\mu p$ where p is the perpendicular from the origin on the tangent plane. Then the density at any point is $Qp/4\pi abc$. The value of z for a point on the ellipsoid for which $x=3$ and $y=3$, is $\frac{3\sqrt{31}}{20}$ and the equation of the tangent plane is $\frac{3x}{25}+\frac{3y}{16}+\frac{3\sqrt{31}}{9}z=1$. The perpendicular from $(0, 0, 0)$ is $\frac{D}{\sqrt{(A^2+B^2+C^2)}}$ or $4.2+$. Therefore the density at $\left(3,\ 3,\ \frac{3\sqrt{21}}{20}\right)=\frac{80.(4.2)}{4\pi(5.4.3)}$ $=.445+$. If the density of this part be a,

$$Q=\frac{4\pi a(5.4.3)}{4.2+}=\frac{240\pi a}{4.2+}.$$

Also solved by G. B. M. Zerr, and the Proposer.

153. Proposed by CHRISTIAN HORNUNG, A. M., Heidelberg University, Tiffin, Ohio.

Two men start from Columbus, Ohio, at the same time; one travels east and the other west. They travel at the rate of 4 miles an hour from sunrise to sunset each day until they meet. Where will they meet and what distance will each have traveled?